Atmospheric coefficient of the planet.

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The minimum possible size of a planet that can support life can be calculated, since gravity depends on the mass and radius of the planet, and holds the atmosphere. It is logical to assume that the planet will support life if it can retain the necessary atmosphere (for example, oxygen).

So, the square of the escape velocity on the surface of the planet is equal to:

$$v^2 = (2 * G * m) / r$$

where G is the gravitational constant,

m is the mass of the planet,

r is the radius of the planet.

Next, the square of the root mean square speed of the gas molecules is (note: the mean square speed of all molecules in the gas is the square root of the arithmetic mean of the squared speeds of each molecule):

$$v^2 = (3 * R * T) / M$$

where R is the universal gas constant,

T – temperature,

M - molar mass (kg/mol).

Now let us assume that the escape velocity on the surface of the planet is equal to the root mean square velocity of the atmospheric molecules. Then, we get the following formula:

$$(2 * G * m) / r = (3 * R * T) / M$$

This is the key formula.

From this formula follows an expression for the square of the radius of the planet, depending on the density of the planet and the temperature of the atmosphere, as well as on the type of gas molecules:

$$r^2 = (9 * R * T) / (8 * \pi * G * \rho * M)$$

where ρ is the density of the planet.

For further discussion, it is necessary to introduce the concept of atmospheric coefficient.

Atmospheric coefficient (AC, n) is the ratio of the escape velocity on a given planet to the root mean square velocity of atmospheric molecules.

Theoretically, there should be a certain minimum value of the coefficient, which in an integrated form will take into account all the factors influencing the preservation of the planet's atmosphere (temperature gradient from altitude, distribution of gas molecules by speed, gravitational field gradient, etc.). That is, at a certain value of the atmospheric coefficient (AC), the planet can be guaranteed to maintain a dense atmosphere like Earth.

Entering the atmospheric coefficient will transform the formula to:

$$r^2 = n^2 * (9 * R * T) / (8 * \pi * G * \rho * M)$$

Taking into account the constants, we obtain the final formula for the calculation:

$$r^2 = n^2 * 4.46099 * 10^{10} * T / (\rho * M)$$

where n is the atmospheric coefficient, T is temperature (˚K),

ρ is the density of the planet, M is the molar mass (kg/mol).

According to the rule of thumb, we assume that the minimum coefficient at which the planet retains its atmosphere is six [1].

It is important to note that at low values of the atmospheric coefficient (6, 8, 10), the gravitational influence of tidal forces will be small, and therefore, a dense, homogeneous atmosphere will extend to a fairly significant height and have a large volume. This means that the biosphere of such a planet will not be able to produce a sufficient amount of biogenic gases (for example, oxygen), which must first fill the atmosphere and then take part in the gas cycle on the planet. Consequently, on planets with a low atmospheric coefficient, the existence of an atmosphere is possible (the planet can retain an atmosphere), but biological life with highly organized organisms is impossible.

If the atmospheric coefficient on the planet is 20 or more, then the gravitational influence of the planet's tidal forces on the atmosphere will be noticeable, and as a result, the atmosphere will be pressed to the surface of the planet by a thin ball (as on Earth) and will have a small height.

For example, the Earth's atmosphere extends up to 100 km (AC = 23.13), but 90 % of the entire atmosphere is within 16 km of the surface. The atmosphere of Venus extends upward to 250 km (AC = 16.03), but 90 % of the entire atmosphere is within 28 km of the surface [2].

Titan's atmosphere is about 400 km thick [3], of course, with a lower coefficient (AC = 9.14). Given the small atmospheric coefficient, Titan's atmosphere should be more uniform in height than that of Earth or Venus.

At large atmospheric coefficients (AC = 20 or more), the biosphere will already be able to fill the planet's atmosphere with oxygen, since the troposphere will have a small thickness and contain 80 – 90 % of the atmosphere. And further, with the participation of the biosphere, the circulation of biogenic gases on the planet (oxygen, carbon dioxide, etc.) will begin to occur, which will significantly increase the rate of evolution of living systems.

To compare sizes, let's calculate the radii of the planets (in terms of oxygen) depending on the density of the planet and the atmospheric coefficient, while assuming that life will be similar to that on Earth (oxygen, water, temperature, etc.). We also assume that the atmospheric temperature is 300 ˚K.

1. Let's determine the radius based on the density of the Earth (ρ = 5.5153 * 10^3 kg/m^3). The radius of the Earth is 6371.0 km.

Then the calculated radius of the planet is:

n = 6, r = 1652.22 km;

n = 9.14 (Titan), r = 2516.88 km;

n = 16.03 (Venus), r = 4414.18 km;

n = 23.13 (Earth), r = 6369.31 km.

2. Let's determine the radius based on the density of Venus (ρ = 5.24 * 10^3 kg/m^3). The radius of Venus is 6051.8 km.

Then the calculated radius of the planet is:

n = 6, r = 1695.07 km;

n = 9.14 (Titan), r = 2582.15 km;

n = 16.03 (Venus), r = 4528.65 km;

n = 23.13 (Earth), r = 6534.48 km.

3. Let's determine the radius based on the density of Mars (ρ = 3.933 * 10^3 kg/m^3). The radius of Mars is 3389.5 km.

Then the calculated radius of the planet is:

n = 6, r = 1956.55 km;

n = 9.14 (Titan), r = 2980.47 km;

n = 16.03 (Venus), r = 5227.24 km;

n = 23.13 (Earth), r = 7542.49 km.

4. Let's determine the radius based on the density of Titan (ρ = 1.8798 * 10^3 kg/m^3). The radius of Titan is 2576 km.

Then the calculated radius of the planet is:

n = 6, r = 2830.07 km;

n = 9.14 (Titan), r = 4311.13 km;

n = 16.03 (Venus), r = 7560.99 km;

n = 23.13 (Earth), r = 10909.91 km.

5. Let's determine the radius based on the Kepler – 10 b density (ρ = 8.8 * 10^3 kg/m^3).

Then the calculated radius of the planet is:

n = 6, r = 1308.01 km;

n = 9.14 (Titan), r = 1992.54 km;

n = 16.03 (Venus), r = 3494.57 km;

n = 23.13 (Earth), r = 5042.38 km.

Thus, the minimum radius of a planet (n = 6) that can maintain an atmosphere at a comfortable temperature (300 ˚K) will be in the range of 1308.01 - 2830.07 km, that is, in the range of 0.75 - 1.63 radius of the Moon (1737.1 km).

With a large atmospheric coefficient (n = 23.13; Earth), the radius of the planet will be in the range of 5042.38 - 10909.91 km, that is, within 0.79 - 1.71 radii of the Earth (6371 km).

Now, for clarity, let's do a small calculation.

Considering that Titan's atmosphere consists of nitrogen (98.4 %) and methane (1.6 %), and the surface temperature is 93.7 ˚K (−179.5 °C), we calculate the radius of Titan by nitrogen at different atmospheric coefficients at a temperature of 93.7 ˚K. According to the calculation, the radius is equal to:

n = 6, r = 1690.84 km;

n = 9.14 (Titan), r = 2575.71 km;

n = 16.03 (Venus), r = 4517.35 km;

n = 23.13 (Earth), r = 6518.18 km.

Similarly, let's calculate the radius of Venus - the calculation will be carried out using carbon dioxide and at a temperature of 737 ˚K, since the atmosphere of Venus consists of carbon dioxide (96.5 %) and nitrogen (3.5 %), and the surface temperature is 737 ˚K (464 °C). According to the calculation, the radius is:

n = 6, r = 2265.73 km;

n = 9.14 (Titan), r = 3451.47 km;

n = 16.03 (Venus), r = 6053.28 km;

n = 23.13 (Earth), r = 8734.40 km.

As you can see, we got good correlations.

1. Quora: Dave Kimber's answer to Does a bigger planet command bigger sized life forms? Or smaller ones?
2. Atmosphere of Venus - Wikipedia.
3. Atmosphere of Titan - Wikipedia. Wikipedia (ru).